

9 MATH

Quarter 1



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This module is a resource of information and guide in understanding the Most Essential Learning Competencies (MELCs). Understanding the target contents and skills can be further enriched thru the K to 12 Learning Materials and other supplementary materials such as worksheets/activity sheets provided by schools and/or Schools Division Offices and thru other learning delivery modalities including radio-based and TV-based instruction (RB/TVI).

CLMD CALABARZON

Mathematics

Grade 9

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Mathematics Grade 9
PIVOT IV-A Learner's Material
Quarter 1 Module 1
First Edition, 2020

Published by: Department of Education Region IV-A CALABARZON
Regional Director: Wilfredo E. Cabral
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Guide in Using PIVOT Learner's Material

For the Parents/Guardian

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners in guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

As a facilitator, you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are

For the Learner

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will have to process the contents of the learning resource while being an active learner.

PARTS OF PIVOT LEARNER'S MATERIAL

	Parts of the LM	Description
Introduction	What I need to know	The teacher utilizes appropriate strategies in presenting the MELC and desired learning outcomes for the day or week, purpose of the lesson, core content and relevant samples. This allows teachers to maximize learners awareness of their own knowledge as regards content and skills required for the lesson
	What is new	
Development	What I know	The teacher presents activities, tasks , contents of value and interest to the learners. This shall expose the learners on what he/she knew, what he /she does not know and what she/he wanted to know and learn. Most of the activities and tasks must simply and directly revolved around the concepts to develop and master the skills or the MELC.
	What is in	
	What is it	
Engagement	What is more	The teacher allows the learners to be engaged in various tasks and opportunities in building their KSA's to meaningfully connect their learnings after doing the tasks in the D. This part exposes the learner to real life situations /tasks that shall ignite his/ her interests to meet the expectation, make their performance satisfactory or produce a product or performance which lead him/ her to understand fully the skills and concepts .
	What I can do	
	What else I can do	
Assimilation	What I have learned	The teacher brings the learners to a process where they shall demonstrate ideas, interpretation , mindset or values and create pieces of information that will form part of their knowledge in reflecting, relating or using it effectively in any situation or context. This part encourages learners in creating conceptual structures giving them the avenue to integrate new and old learnings.
	What I can achieve	

Quadratic Equations

Lesson

After going through this lesson, you are expected to:

- Illustrates quadratic equations.
- Solves quadratic equations by:
 - (a) extracting square roots; (c) completing the square; and
 - (b) factoring; (d) using the quadratic formula.

You learned about linear equation in one variable which is in the form of $ax + b = 0$, where a is not 0 otherwise the equation is constant. Quadratic equation is in the form of $ax^2 + bx + c = 0$ and a cannot be zero otherwise the equation will become linear equation.

Learning Task 1. Group the given equations into two based on observed common properties.

$n^2 - 3n + 10 = 0$	$8 - 3k = 12$	$2y - z = 9$	$2x^2 + 2x + 1 = 0$
$25b^2 - 16 = 0$	$3r + 2e = -6$	$5w + 5 = 0$	$f^2 - 3f + 2 = 0$
$d = 3e - 7$	$\frac{1}{3}m^2 + 2m = 4$	$10u - 5 = 8$	$a^2 = 225$

D

The standard form of quadratic equation is $ax^2 + bx + c = 0$, where a , b and c are real numbers and a is not equal to zero.

Illustrative Examples:

1. $2x + 5 = 0$ is not a quadratic equation in one variable. It is a linear equation in one variable.
2. $2x^2 - x - 1 = 0$ is a quadratic equation in standard form with $a = 2$, $b = -1$, and $c = -1$.
3. $3x - 4 = 5x^2$ is a quadratic equation not in standard form, in this case we need to express it in its standard form to identify the values of a , b and c . To write it:

$3x - 4 - 5x^2 = 5x^2 - 5x^2$	(Subtraction Property of Equality)
$3x - 4 - 5x^2 = 0$	(Arrange the terms)
$(-5x^2 + 3x - 4 = 0) - 1$	(Obtain a > 0 , by multiplying -1 to each term of the equation.)

$$5x^2 - 3x + 4 = 0$$

In this form, $a = 5$, $b = -3$ and $c = 4$.

4. $(2x + 3)(x - 1) = 0$ is also a quadratic equation but is not written in standard form.

Expanding :

$$2x(x - 1) + 3(x - 1) = 0$$

$$2x^2 - 2x + 3x - 3 = 0 \quad (\text{Distributive Property})$$

$$2x^2 + x - 3 = 0 \quad (\text{Combining similar terms})$$

In this form, $a = 2$, $b = 1$ and $c = -3$.

A **quadratic equation** in one variable is a mathematical sentence of degree two that can be written in the following standard form: $ax^2 + bx + c = 0$ where, a , b and c are real numbers $a \neq 0$. In this equation, ax^2 is the quadratic term (degree two), bx is the linear term (degree one) and c is the constant term (degree zero).

When $b = 0$, in the equation $ax^2 + bx + c = 0$, the result is a quadratic equation of the form $ax^2 + c = 0$. For example: $x^2 - 16 = 0$, $25x^2 - 81 = 0$ and $5x^2 = 500$. Furthermore, when $c = 0$, the quadratic equation is reduced to $ax^2 + bx = 0$. That is, in $x^2 + 3x = 0$ and $5x^2 - x = 0$, there is no constant term, $c = 0$.

In solving quadratic equation, we can apply the following methods:

A. Solving quadratic equations by extracting square roots.

Remember when it was mentioned that a quadratic equation of the form $ax^2 + bx + c = 0$ may have $b = 0$, such that, $ax^2 + c = 0$. In other words, since c could be any constant, then, $ax^2 = -c$. And, if $a = 1$, the quadratic equation is further reduced to $x^2 = -c$. Recall square roots.

In order to solve a quadratic equation by extracting square roots, the equation must be written in the form $x^2 = c$, before extracting the square roots of the left and right sides of the said equation so as to have the equation balanced. Inspect the given examples.

ILLUSTRATIVE EXAMPLES:

Solving for the values of the variable x by extracting square roots:

- $x^2 = 9$ Quadratic equation in the form $x^2 = c$
 $\sqrt{x^2} = \sqrt{9}$ Extracting the square roots of the left and right sides
 $x = \pm 3$ Possible values that will satisfy the quadratic equation

To check for the solved values, substitute both $+3$ and -3 in the given equation. Also, always remember that a negative number does not have a square root.

2. $x^2 - 25 = 0$ Quadratic equation in the form $x^2 + c = 0$
 $x^2 = 25$ by Addition Property of Equality
 $\sqrt{x^2} = \sqrt{25}$ Extracting the square roots of the left and right sides
 $x = \pm 5$ Possible values that will satisfy the quadratic equation
3. $4x^2 = 49$ Quadratic equation in the form $ax^2 = c$
 $x^2 = \frac{49}{4}$ by Multiplication Property of Equality
multiplying both sides by $\frac{1}{4}$
 $\sqrt{x^2} = \sqrt{\frac{49}{4}}$ Extracting the square $\frac{1}{4}$ roots of the left and right sides
 $X = \pm \frac{7}{2}$ Possible values that will satisfy the quadratic equation

4. Find the roots of the equation $(x - 1)^2 = 0$.

Again, applying extracting square roots:

$$\sqrt{(x-1)^2} = \sqrt{0} \quad \text{Extracting square roots of the left and right sides}$$

$$x - 1 = 0$$

$$x = 1 \quad \text{by Addition Property of Equality}$$

B. Solving quadratic equations by factoring

Only quadratic equation that is factorable can be solved by factoring. To solve such quadratic equation, the following procedure can be followed.

1. Transform the quadratic expression into standard form if necessary.
2. Factor the quadratic expression.
3. Apply zero product property by setting each factor of the quadratic expression equal to 0.
4. Solve the resulting equation.
5. Check the values of the variable obtained by substituting each in the original equation.

Zero Product Property. If a and b, are real numbers, then $(a)(b) = 0$, such that, $a = 0$ or $b = 0$ or a and b are both equal to zero.

So, if a given quadratic equation is in the form $(a)(b) = 0$, the Zero Product Property can be applied. To do this, the given quadratic equation must be written in the standard form $ax^2 + bx + c = 0$ before applying the factoring method.

ILLUSTRATIVE EXAMPLES:

Solve each equation by factoring.

1. $n^2 + 2n + 1 = 0$	Quadratic equation in standard form
$(n + 1)(n + 1) = 0$	Factoring the left side of the equation
$\frac{n + 1 = 0}{n = -1} \quad \frac{n + 1 = 0}{n = -1}$	by Zero Product Property by Addition Property of Equality (APE)

2. $m^2 + 3m + 2 = 0$	Quadratic equation in standard form
$(m + 2)(m + 1) = 0$	Factoring the left side of the equation
$\frac{m + 2 = 0}{m = -2} \quad \frac{m + 1 = 0}{m = -1}$	by Zero Product Property by Addition Property of Equality

C. Solving quadratic equations by completing the square.

If the first two methods in solving quadratic equations cannot be used to solve such quadratic equations, then, you must need another method to solve the said equations. This other way of solving quadratic equations is actually referred to as the mother of all methods in solving any quadratic equation – the completing the square method.

The completing the square method also includes the use of extracting square roots after the completing of square part. You may use a scientific calculator in writing the approximate value/s of the answer/s if they are irrational number values.

Completing the square includes the following steps:

1. Divide both sides of the equation by “a” then simplify.
2. Write the equation such that the terms with variables are on the left side of the equation and the constant term is on the right side.
3. Add the square of one-half of the coefficient of “x” on both sides of the resulting equation. The left side of the equation becomes a perfect square trinomial.
4. Express the perfect square trinomial on the left side of the equation as a square of a binomial.
5. Solve the resulting quadratic equation by extracting the square root.
6. Solve the resulting linear equations.
7. Check the solutions obtained against the original equation.

ILLUSTRATIVE EXAMPLES:

Express the following as a squared binomial by completing the square.

1. $x^2 + \underline{\quad} + 9$

Incomplete perfect square trinomial (the quadratic/ first and constant/last terms are perfect squares)

$$x^2 + \underline{6x} + 9$$

The middle or linear term is found by doubling the product of the square roots of the first and last terms

$$2 \cdot \sqrt{x^2} \cdot \sqrt{9}$$

$$(x + 3)(x + 3)$$

Writing as product of the same binomial or as a

$$(x + 3)^2$$

Squared binomial

2. $4e^2 - \underline{\quad} + 25$

Incomplete perfect square trinomial (the quadratic/ first and constant/last terms are perfect square)

$$4e^2 - \underline{20e} + 25$$

The middle or linear term is found by doubling the product of the square roots of the first and last terms

$$2 \cdot \sqrt{4e^2} \cdot \sqrt{25}$$

$$(2e - 5)(2e - 5)$$

Writing as product of the same binomial or as

$$(2e - 5)^2$$

Squared binomial

NOTE: The sign of the middle (missing) term will also be the sign of the operation between the two terms in the squared binomial.

What do you do when the quadratic trinomial to be completed does not seem to be incomplete – it consists of three terms but is not a perfect square trinomial?

3. Express $x^2 + 2x + 4$ as a squared binomial by completing the square

$$x^2 + 2x + 4$$

Quadratic but not a perfect square trinomial

$$x^2 + 2x + \underline{\quad} + 4 - \underline{\quad}$$

Terms to be added must sum up to zero

$$x^2 + 2x + \underline{1} + 4 - \underline{1}$$

The added term is the square of half the numerical coefficient of the middle term

$$x^2 + 2x + 1 + 3$$

Combining constants

$$(x + 1)^2 + 3$$

Writing the perfect square trinomial as a squared binomial

D. Solving quadratic equations using the quadratic formula.

For any given quadratic equation (in one variable) in the standard form $ax^2 + bx + c = 0$, all you need to do is substitute the corresponding values of the numerical coefficients a , b and c from the standard form of the quadratic equation in the formula;

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ILLUSTRATIVE EXAMPLES:

1. $x^2 + 6x + 5 = 0$, the values of $a = 1$, $b = 6$ and $c = 5$.

Then, using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 20}}{2}$$

$$x = \frac{-6 \pm \sqrt{16}}{2}$$

$$x = \frac{-6 \pm 4}{2}$$

$$x = -1 \text{ or } -5 \quad (\text{Check the solutions/roots by substituting these to the original quadratic equation})$$

Note: Quadratic equation has at most two zeros or roots.

Learning Task 2

A. Complete the table below

Given	Standard Form	Values of		
		a	b	c
1. $2x - 3x^2 = 5$				
2. $4 - x^2 = 5x$				
3. $(2x + 5)(x - 4) = 0$				
4. $2x(x - 1) = 6$				
5. $(x + 1)(x + 4) = 8$				

B. Solve the quadratic equation using appropriate method.

1. $x^2 - 81 = 0$ 2. $x^2 + 5x + 6 = 0$ 3. $2x^2 - 4x + 3 = 0$

E

Learning Task 3 . Solve for the variable of the following quadratic equations

A. by extracting square roots.

1. $x^2 = 169$

4. $(x - 2)^2 = 16$

2. $9b^2 = 25$

5. $2(t - 3)^2 - 72 = 0$

3. $(3y - 1)^2 = 0$

B. by factoring

1. $x^2 + 7x = 0$

3. $x^2 + 5x - 14 = 0$

2. $m^2 + 8m = -16$

4. $2y^2 + 8y - 10 = 0$

C. by completing the square.

1. $x^2 + 5x + 6 = 0$

2. $x^2 + 2x = 8$

3. $2x^2 + 2x = 24$

D. using quadratic formula.

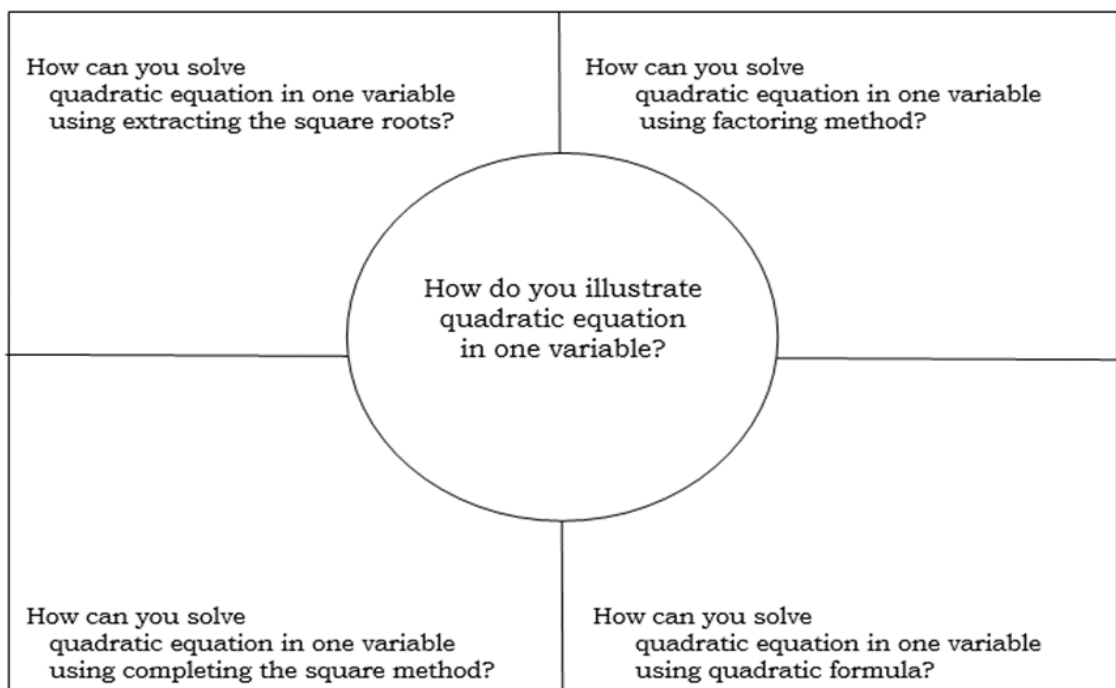
1. $x^2 + 5x = 14$

2. $2x^2 + 8x - 10 = 0$

3. $2x^2 + 3x = 27$

A

Learning Task 3. Using the concept map below explain what you have learned in this module.



Characterizing and Describing the Roots of Quadratic Equations

WEEKS

2-3

I

Lesson

After going through this lesson, you are expected to:

- Characterize the roots of a quadratic equation using the discriminant.
- Describe the relationship between the coefficients and the roots of a quadratic equation.

Roots of quadratic equation can be imaginary number, equal or two distinct roots. It can be determined by the value of $b^2 - 4ac$.

Learning Task 1. Complete the table given below and find the relations among constants a , b and c in the quadratic equation standard form.

Equation	a	b	c	$b^2 - 4ac$	Roots	
					x_1	x_2
$x^2 + 4x + 3 = 0$						
$x^2 - 5x + 4 = 0$						
$x^2 - 49 = 0$						
$4x^2 - 25 = 0$						
$2x^2 + 7x + 3 = 0$						

D

The expression $b^2 - 4ac$, is the quadratic equation's discriminant. The discriminant determines the nature of the roots of a quadratic equation.

If $b^2 - 4ac = 0$, then the roots are real and equal

$b^2 - 4ac > 0$, then the roots are real and unequal

$b^2 - 4ac < 0$, then the roots are not real

Illustrative Examples:

1. Find the nature of roots of the equation $x^2 + 4x + 3 = 0$.

The values of a , b and c in the equation are 1, 4 and 3, respectively.

Evaluating $b^2 - 4ac$,

$$\begin{aligned}b^2 - 4ac &= (4)^2 - 4(1)(3) \\ &= 16 - 12 \\ &= 4\end{aligned}$$

Since $b^2 - 4ac > 0$, then the equation has two real and unequal roots.

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To check, solve for the roots of the equation.

Checking,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 12}}{2}$$

$$x = \frac{-4 \pm 2}{2}$$

$$x = -1 \text{ or } -3 \text{ (rational and unequal)}$$

When the value of the determinant $b^2 - 4ac$ is greater than zero and is not a perfect square, then the roots are irrational numbers and unequal.

2. Determine the nature of roots of the equation $x^2 - 6x + 9 = 0$.

The values of a, b and c in the equation are 1, -6 and 9, respectively.

Evaluating $b^2 - 4ac$,

$$\begin{aligned} b^2 - 4ac &= (-6)^2 - 4(1)(9) \\ &= 36 - 36 \\ &= 0 \end{aligned}$$

Since $b^2 - 4ac = 0$, then the roots of quadratic equation are real and equal.

Check: $x^2 - 6x + 9 = 0$

$$(x - 3)^2 = 0 \longrightarrow x - 3 = 0 \longrightarrow x = 3$$

3. What kind of roots does the equation $x^2 + 2x + 3 = 0$ have?

The values of a, b and c in the equation are 1, 2 and 3, respectively.

Evaluating $b^2 - 4ac$,

$$\begin{aligned} b^2 - 4ac &= (2)^2 - 4(1)(3) \\ &= 4 - 12 \\ &= -8 \end{aligned}$$

Since $b^2 - 4ac < -8$, then the roots are not real.

The sum and product of the roots of a given quadratic equation (in one variable) as you have noticed in the activity table have relations among the constants a, b and c of said equation in standard form. This is given by the actual sum and product of the roots in using the Quadratic Formula to solve a given quadratic equation: given that the roots are:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

<p>Sum of the Roots = $x_1 + x_2$</p> $= \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-b - b}{2a}$ $= \frac{-2b}{2a}$ $= -\frac{b}{a}$ <p>$x_1 + x_2 = -\frac{b}{a}$</p>	<p>Product of the Roots = $x_1 \cdot x_2$</p> $= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \cdot \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ $= \frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{4a^2}$ $= \frac{b^2 - b^2 + 4ac}{4a^2}$ $= \frac{4ac}{4a^2}$ $= \frac{c}{a}$ <p>$x_1 \cdot x_2 = \frac{c}{a}$</p>
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So, the sum of the roots of a given quadratic equation is the ratio $-\frac{b}{a}$,

while the product of its roots is the ratio $\frac{c}{a}$. In general, all you need to do is

write the quadratic equation in standard form and identify the values of the constants a, b and c. In finding the sum and product of the roots of the equations:

a. $x^2 + 5x + 4 = 0$, the values of $a = 1$, $b = 5$ and $c = 4$, respectively.

Substituting values in the ratio $-\frac{b}{a}$ for

Sum of roots, $x_1 + x_2 = -\frac{b}{a} = -\frac{5}{1} = -5$, while for

Product of roots, $x_1 \cdot x_2 = \frac{c}{a} = \frac{4}{1} = 4$.

Checking:

Using factoring, $(x + 4)(x + 1) = 0$ Factoring the quadratic trinomial part

$x + 4 = 0$; $x + 1 = 0$ Zero Product Property

$x = -4$; $x = -1$ by APE

$x_1 + x_2 = -4 - 1 = -5$ Sum of roots

$x_1 \cdot x_2 = (-4)(-1) = 5$ Product of roots

b. $2x^2 - 5x + 3 = 0$, the values of $a = 2$, $b = -5$ and $c = 3$, respectively.

Substituting values in the ratio $-\frac{b}{a}$ for

Sum of roots, $x_1 + x_2 = -\frac{b}{a} = -\frac{5}{2}$, while for

Product of roots, $x_1 \cdot x_2 = \frac{c}{a} = \frac{3}{2}$.

Checking:

Using factoring, $(2x - 3)(x - 1) = 0$ Factoring the quadratic trinomial part

$2x - 3 = 0$; $x - 1 = 0$ Zero Product Property

$2x = 3$; $x = 1$ by APE

$x = \frac{3}{2}$ by MPE

Learning Task 2. Complete the table

Equation	a	b	c	Discriminant	Nature of the Roots
$x^2 - 6x - 27 = 0$					
$x^2 - 25 = 0$					
$x^2 + 10x + 25 = 0$					
$2x^2 - 5x + 3 = 0$					

E

Learning Task 3. Do the following:

A. Characterize the roots of the following quadratic equations using the discriminant.

1. $x^2 + 4x + 3 = 0$

4. $4x^2 - 4x + 1 = 0$

2. $x^2 - 5x + 4 = 0$

5. $2x^2 + 6x + 3 = 0$

3. $x^2 + 7 = 0$

B. Complete the table.

Equation	a	b	c	Roots		$x_1 + x_2$	$x_1 \cdot x_2$
				x_1	x_2		
$x^2 + 5x + 4 = 0$							
$x^2 - 6x - 27 = 0$							
$x^2 - 25 = 0$							
$x^2 + 10x + 25 = 0$							
$2x^2 - 5x + 3 = 0$							

A

Learning Task 4. Solve the problem by applying the sum and product of roots of quadratic equations.

The perimeter of a rectangular metal plate is 36 dm and its area is 80 dm². Find its dimensions. (Relate the measures to the sum and product of a quadratic equation.)

The perimeter of a rectangle is twice the sum of its length and width while its area is the product of its length and width. Such that,

Perimeter = $2(L + w)$ and Area = $L \cdot w$

Solving Quadratic Equations and Rational Algebraic Equations

WEEKS

4-5

I

Lesson

After going through this lesson, you are expected to:

- Solve equations transformable to quadratic equations (including rational algebraic equations).
- Solve problems involving quadratic equations and rational algebraic equations.

So far, what you have solved are quadratic equations in standard form or one that can be written in standard form. What if the quadratic equation involves rational expressions? Can one also use the learned methods to solve such equations?

Since, the quadratic equations this time involves rational expressions, you must recall the concept on finding the least common denominator of rational expressions.

Learning Task 1. Find the least common denominators of the following expressions:

1. $\frac{1}{2} + \frac{3}{4}$ 3. $\frac{2}{x} - \frac{x+1}{5}$

2. $\frac{1}{x} + \frac{x}{2}$ 4. $\frac{2}{x-3} + \frac{x}{2}$

What about the rational expressions in each equation, what should be the corresponding least common denominator?

1. $\frac{1}{x} + \frac{x}{2} = \frac{11}{6}$

2. $\frac{2}{x} - \frac{x+1}{5} = -\frac{4}{5}$

3. $\frac{2}{x-3} + \frac{x}{2} = -\frac{1}{2}$

D

In order to simplify a rational equation, it has to be transformed into an equation without the denominators. This is made possible by using the least common denominator of all rational expressions in the equation containing such expressions.

ILLUSTRATIVE EXAMPLES:

1. Solve the rational equation $\frac{x}{3} + \frac{1}{x} = \frac{19}{12}$ by transforming it into a quadratic equation. The least common denominator (LCD) of the left rational parts in the equation is $(3)(x)$. The LCD is generally found by multiplying all the denominators – as long as they are relatively prime (no other factor will divide the expressions except 1).

Then, $\frac{x^2 + 3}{3x} = \frac{19}{12}$	by Addition (using the LCD on the left side)
$12(x^2 + 3) = (3x)(19)$	by Proportionality
$12x^2 + 36 = 57x$	Distributive Property
$12x^2 - 57x + 36 = 0$	by APE
$4x^2 - 19x + 12 = 0$	by MPE
$(4x - 3)(x - 4) = 0$	by factoring method
$4x - 3 = 0 \quad \Bigg \quad x - 4 = 0$	Zero Product Property
$4x = 3 \quad \Bigg \quad x = 4$	by APE
$x = \frac{3}{4} \quad \Bigg \quad$	by MPE

To transform rational equations into quadratic equations, the following procedures can be followed:

1. Multiply both sides of the equation by the LCD (Least Common Denominator).
2. Write the resulting quadratic equation in standard form.

Then, you can solve the resulting quadratic equation using any methods of solving quadratic equation. And don't forget to check the obtained values by substituting in the original equation.

So far, how did you find solving some of the problem applications regarding the different concepts on quadratic equations? Let us now have a deeper understanding of some other problem conditions involving solving quadratic and rational equations.

ILLUSTRATIVE EXAMPLES:

1. Odette can finish washing a certain number of dishes in 15 minutes less than it takes Pam. If they work together, they can do the dishes in 10 minutes.

Let $w = (1)$ whole work to be done

$$\frac{1}{t} = \text{part of work done by Pam in a minute}$$

$\frac{1}{t-15}$ = part of work done by Odette in a minute

$\frac{1}{10}$ = part of work done together by both girls in a minute

Equation: $\frac{1}{t} + \frac{1}{t-15} = \frac{1}{10}$

Find the LCD, Then add: $\frac{1}{t} + \frac{1}{t-15} = \frac{1}{10} \rightarrow \frac{(t-15) + t}{t(t-15)} = \frac{1}{10}$

$$\frac{2t-15}{t^2-15t} = \frac{1}{10} \rightarrow 10(2t-15) = t^2-15$$

$$20t - 150 = t^2 - 15 \rightarrow t^2 - 20t + 145 = 0$$

You may solve the equation by any method applicable.

Learning Task 2.

A. Translate the following verbal sentences to mathematical sentence. Then express into quadratic equations in terms of “x”.

Given	Quadratic Equations
1. The length of a wooden frame is 1 foot longer than its width and its area is equal to 12 ft ² .	
2. The length of the floor is 8 m longer than its width and there is 20 square meters.	
3. The length of a plywood is 0.9 m more than its width and its area is 0.36 m ² .	
4. The area of rectangle whose length is six less than twice its width is thirty-six.	
5. The width of a rectangular plot is 5 m less than its length and its area is 84 m ² .	

B. Solve for x:

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{10}$$

E**Learning Task 3**

A. Solve the following equations transformable to quadratic equations.

1. $\frac{1}{x} + \frac{x}{2} = \frac{11}{6}$	2. $\frac{2}{x} - \frac{x+1}{5} = -\frac{4}{5}$	3. $\frac{2}{x-3} + \frac{x}{2} = -\frac{1}{2}$
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B. Solve

1. A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more it would have taken 30 minutes less for the journey. Find the original speed of the bus.
2. If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number.

A**Learning Task 4**

Solve the following problems. Show the step by step process. And write a brief explanation of what you do.

1. The width of a bedroom floor is 5 m less than its length and its area is 84 m^2 . Find its dimensions.
2. A certain pipe can fill up a tank 2 hours faster than another pipe. It takes 3 and hours for both pipes to fill up the same tank. In how many hours would the first pipe fill up the tank?
3. A picture frame has its length 8 cm longer than its width. It has an inner 1-cm boundary such that a maximum 660 cm^2 -picture may fit into it. Find the dimensions of this frame.
4. After how many seconds is needed for a thrown ball to reach a 14-m distance if its distance is given by the relation $d = 10 + 40t - 5t^2$? (t =time in seconds).

Quadratic Inequalities

Lesson

I

After going through this lesson, you are expected to:

- Illustrates quadratic inequalities.
- Solves quadratic inequalities.
- Solves problems involving quadratic inequalities.

Learning Task 1. Complete the table below by identifying whether the given is quadratic inequality or not. Put a check mark (✓) in the column of your choice. Then answer the questions that follows.

Given	Quadratic Inequality	Not
1. $x^2 - x - 6$		
2. $3x^2 - 4x - 7 = 0$		
3. $(x - 4)(x - 2) > 0$		
4. $x^2 - 8x - 16 \leq 0$		
5. $2x^2 - 5x - 10 > 0$		

D

A quadratic inequality is mathematical statement that relates a quadratic expression as either less than or greater than another.

Illustrative Examples:

1. $3x^2 - 6x - 2$; this is not quadratic inequality in one variable because there is no inequality symbol indicated.
2. $x^2 - 2x - 5 = 0$; this is not quadratic inequality in one variable because symbol used is equal sign. It is quadratic equation in one variable.
3. $x - 2x - 8 \geq 0$; this is not quadratic inequality in one variable because the highest exponent is not 2.
4. $(x - 5)(x - 1) > 0$; this is quadratic inequality in one variable because the symbol used is an inequality symbol and the highest exponent is 2. (But take note this one is tricky because you need to multiply the factors to expressed to its lowest term then can identify the highest exponent. Thus,

$$(x - 5)(x - 1) > 0 \quad \text{using FOIL Method}$$

$$(x - 5x - x + 5) > 0 \quad \text{Combine similar terms to simplify}$$

Then, $x^2 - 6x + 5 > 0$ is the standard form of the given equation

A **quadratic inequality in one variable** is an inequality that contains polynomial whose highest exponent is 2. The general forms are the following, where a, b, and c are real numbers with $a > 0$ and $a \neq 0$.

$$ax^2 + bx + c > 0$$

$$ax^2 + bx + c < 0$$

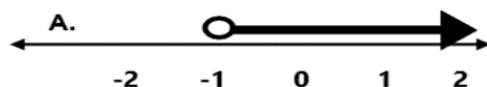
$$ax^2 + bx + c \leq 0$$

$$ax^2 + bx + c \geq 0$$

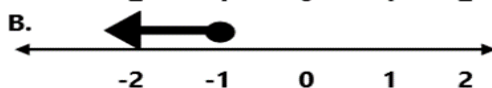
In solving quadratic inequalities, we need to find its solution set. The **solution set** of a quadratic inequality can be written as a set and can be illustrated through a number line.

To illustrate the solution set on the number line, we need to consider the inequality symbols used in given quadratic inequality.

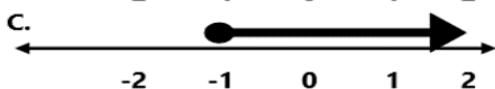
1. $x > -1$



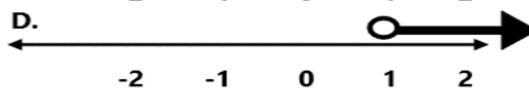
2. $y \leq -1$



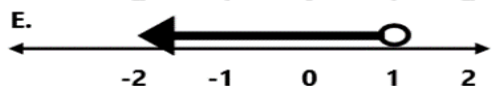
3. $y \geq -1$



4. $x > 1$



5. $x < -1$



SYMBOLS	Circles to be used
<	○ Hollow circle (the critical points are not included)
>	
≤	● Solid circle (the critical points are included)
≥	

Illustrative Examples:

Find the solution set of $3x^2 - 3x - 18 \leq 0$.

$$3x^2 - 3x - 18 \leq 0$$

$$3x^2 - 3x - 18 = 0$$

Transform the given inequality into equation

$$(3x + 6)$$

$$(x - 3) = 0$$

Factor the quadratic expression

$$3x + 6 = 0$$

$$x - 3 = 0$$

Equate each factor by zero

$$3x + 6 = 0$$

$$x - 3 = 0$$

Transpose the constants to the right side

$$3x = -6$$

$$x = 3$$

Solve for x.

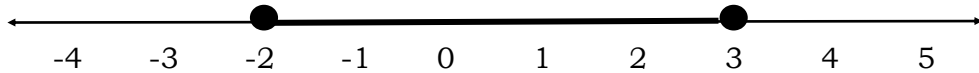
$$x = -2$$

$$x = 3$$

The values of x which are -2 and 3 are the **critical points** which can be used to obtain the following intervals: **$(x \leq -2)$** , **$(-2 \leq x \leq 3)$** , and **$(x \geq -3)$** .

Take note that -2 and 3 are included in the solution set because the symbol used is \leq (less than or equal to).

Therefore, the solution set of the inequality is **$\{-2 \leq x \leq 3\}$** , and its graph is shown below.



To solve quadratic inequalities, the following procedures can be followed.

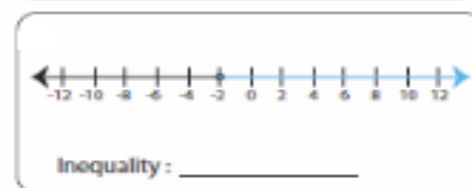
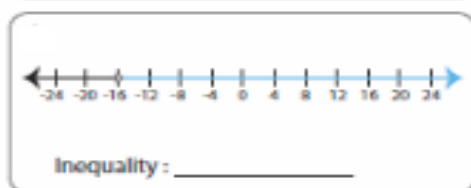
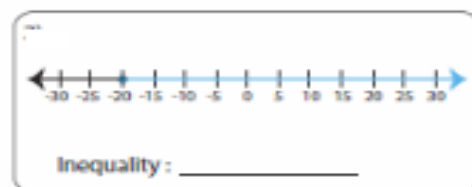
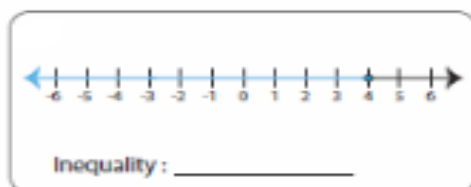
1. Transform the given quadratic inequality into quadratic equation. (Make sure to express into standard form.)
2. Solve for the roots (critical points). You may use the different methods in solving quadratic equations.
3. Use the roots (critical points) to obtain the intervals.
4. Choose a test point in each interval to determine the solution set.
5. Graph the solution set.
6. Check the obtained solution set in the original inequality

Learning Task 2

A. Graph the following on a number line

- | | |
|----------------|---------------|
| 1. $x > 2$ | 4. $x < -4$ |
| 2. $x \geq -3$ | 5. $x \leq 5$ |
| 3. $x > 5$ | |

B. Write the inequality



E

Learning Task 3. Tell whether the given statements below illustrate quadratic inequality in one variable or not. You may translate into mathematical symbols to justify your answers.

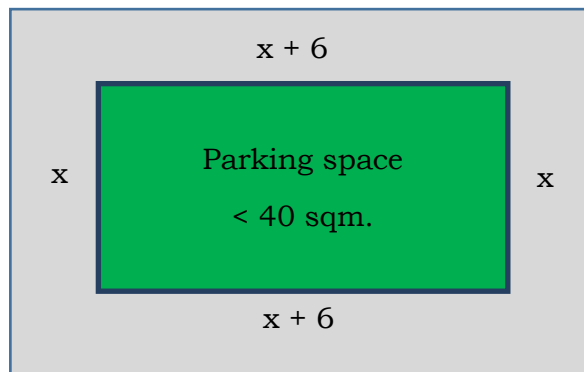
1. Four more than thrice number is greater than forty.
2. The length of the floor is 8 m longer than its width and the area is greater than 20 square meters.
3. The width of a rectangular plot is 5 m less than its length and its area is 84 m².

Steps in Solving Problems involving Inequalities.

1. Read and understand the problem.
2. Identify the given conditions and the unknown.
3. Represent the unknown using variable.
4. Devise an equation or inequality that corresponds to the given conditions and the unknown in the problem.
5. Solve for the unknown,
6. Check your answer.

Solve the problem:

Mrs. Reyna is planning to fence her vacant lot for a parking space. The desired length of the lot is 6 meters longer than its width. What will be the possible dimensions of the rectangular parking space if it should be less than 40sqm? (Hint: length \times width = Area of rectangle)



A

Learning Task 4. Solve the problem below. (Show the step by step process)

The length of the floor is 32 m longer than its width and there is greater than 200 square meters. You will cover the floor completely with tiles. What will be the possible dimension of the floor?

Modeling, Representing and Transforming Quadratic Functions

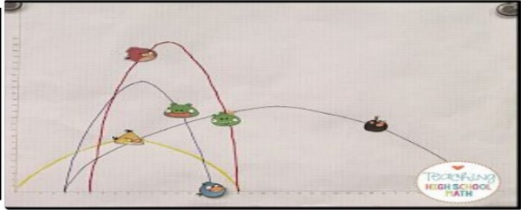

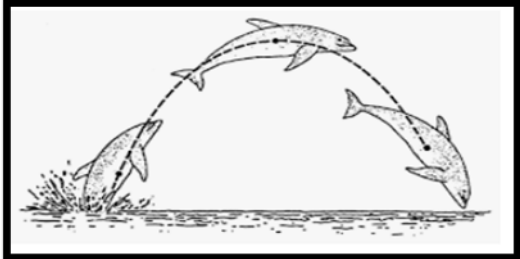
I

After going through this lesson, you are expected to:

- Models real-life situations using quadratic functions.
- Represents a quadratic function using: (a) table of values; (b) graph; and (c) equation.
- Transforms the quadratic function defined by

$$y = ax^2 + bx + c \text{ into the form } y = a(x - h)^2 + k.$$

Learning Task 1. Tell whether each picture models a quadratic function or not. Justify your answer.

GIVEN	Answer	WHY?
 <p>Source: Google search//www.pinterest.ph</p>		
 <p>Source: Google search//ww.coolmath.com</p>		
 <p>Source: Google search//ww.thinglink.com</p>		

D

The concepts of quadratic function is very useful in our life if you know further about it. You can solve different problems that involves quadratic function in real-life situations such as building structures, computing the maximum height or minimum point of an object my reach, analyzing the movement of an object, and etc.

Here are some examples of situations that models quadratic functions in real-life situations.

1. Targets an object in upward direction
2. Throwing an object downward
3. Shooting ball vertically upward
4. Minimum point submarine to submerge
5. Launching rocket to its maximum point

Study the illustrative examples below to know more about how quadratic function.

Illustrative Examples:

1: Is $y = f(x) = x^2 + 6x + 8$ a quadratic function or not?

Solution: $f(x) = x^2 + 6x + 8$ is a quadratic function since its highest degree is 2 and all numerical coefficients are real numbers.

2: Is $y = 7x + 12$ a quadratic function or not?

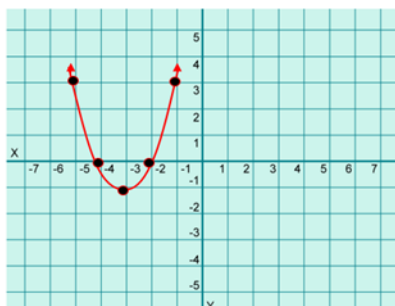
Solution: $y = 7x + 12$ is a not quadratic function since its highest degree is 1.

3: Is $f(x) = (x-2)(x+3)x^2$ a quadratic function or not?

Solution: $y = (x-2)(x+3)x^2$ is a not quadratic function because if you will expand the right side, its highest degree will be 4.

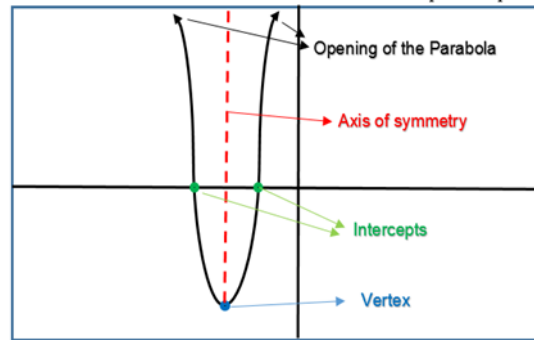
4. Consider the graph of the quadratic function $f(x) = x^2 + 6x + 8$.

x	-5	-4	-3	-2	-1
y= f(x)	3	0	-1	0	3



Observe the trend/ characteristics of the graph for you to determine how quadratic func-

The graph of a quadratic function is a **Parabola**. It has different properties including *vertex*, *axis of symmetry*, *opening of the parabola*, and *the intercepts*. Consider the illustration on the right to



A **quadratic function** is a second degree polynomial represented as $f(x) = ax^2 + bx + c$ or $y = ax^2 + bx + c$, $a \neq 0$ where **a**, **b**, and **c** are real numbers.

The graph of a quadratic function is a **Parabola**. It has different properties including *vertex*, *axis of symmetry*, *opening of the parabola*, and *the intercepts*.

Study the process on how to transform quadratic function defined by $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$.

ILLUTRATIVE EXAMPLES:

1. Transform $y = x^2 - 6x - 6$ into $y = a(x - h)^2 + k$.

$$y = x^2 - 6x - 6$$

$$y = (x^2 - 6x) - 6 \quad \text{Group together the terms containing } x.$$

$$y = (x^2 - 6x + 9) - 6 - 9 \quad \text{Make the expression in parenthesis a perfect square trinomial by adding the value of } \left(\frac{-b}{2}\right)^2 \text{ and subtracting the same value to the constant term, since } a = 1.$$

$$\left(\frac{-6}{2}\right)^2 = 9$$

$$y = (x^2 - 6x + 9) - 15 \quad \text{Simplify.}$$

$$y = (x - 3)^2 - 15 \quad \text{Express the perfect square trinomial into square of binomial}$$

Learning Task 2. Which of the following represents a quadratic function?

1. $f(x) = 8x + 5$ _____

2. $f(x) = x^2 - 2x + 7$ _____

x	-5	-4	-3	-2	-1
y= f(x)	3	0	-1	0	3

3. _____

x	1	2	3	4	5
y= f(x)	0	1	2	3	4

4. _____

5.  _____
6.  _____

B. Quadratic function defined by $y = ax^2 + bx + c$ can be transformed into the form $y = a(x - h)^2 + k$ (called the Vertex Form), by using Completing the Square Method. The following steps can be followed.

1. Group together the terms containing x .
2. Factor out a . If $a = 1$, proceed to step 3.
3. Complete the expression in parenthesis to make it perfect square trinomial by adding the value of $(\frac{b}{2a})^2$ and subtracting the value $a(\frac{b}{2a})^2$ to the constant term.
4. Simplify and express perfect square trinomial as the square of binomial.

E

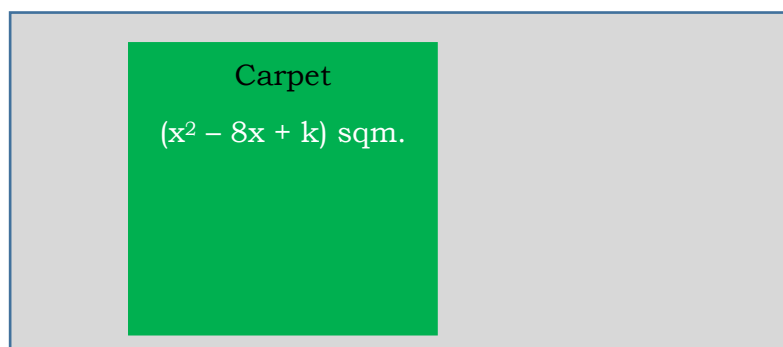
Learning Task 3. Transform the quadratic function defined by $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$.

1. $y = x^2 - 6x - 3$

2. $y = 5x^2 - 20x - 5$

A

Learning Task 4 Suppose that you will put a square carpet on a floor, you need a carpet with an area of $(x^2 - 8x + k)$ square meters. What will be the value of “ k ” so that it will make it a perfect square?



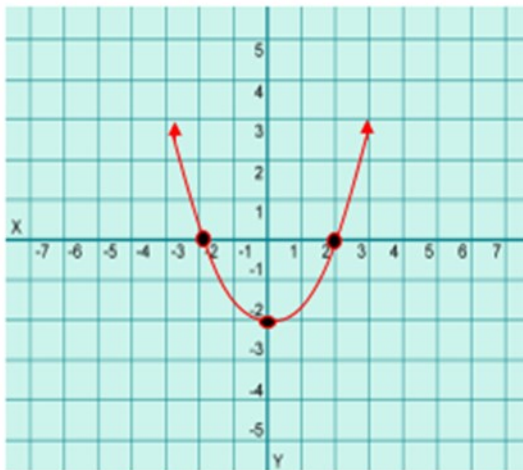
Graphing Quadratic Functions and Analyzing the Effects on its Graph

After going through this lesson, you are expected to:

- Graphs a quadratic function: (a) domain; (b) range; (c) intercepts; (d) axis of symmetry; (e) vertex; (f) direction of the opening of the parabola.
- Analyzes the effects of changing the values of a , h and k in the equation $y = a(x - h)^2 + k$ of a quadratic function on its graph..

In this time, you will learn how to graph quadratic functions and analyze the effects of changing the values of a , h and k in the equation $y = a(x - h)^2 + k$.

Learning Task 1. Complete the table below using the given graph.



Domain	
Range	
Opening of the parabola	
Vertex	
Axis of Symmetry	
x - intercept	
y - intercept	

Learning Task 2. Sketch the graph of the following. Give the domain and range of the following quadratic function defined by the given equation. Determine the direction where the parabola opens, its vertex, its axis of symmetry, and its x- and y- intercepts.

1. $y = x^2 + 8x + 4$

2. $y = x^2 + 4x + 5$

3. $y = -3x^2 + 12x - 7$

4. $y = 2x^2 - 4x - 6$

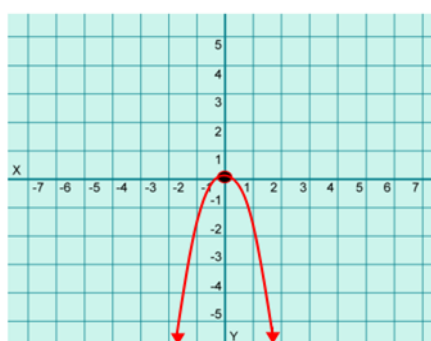
Take note that we have two kinds of parabola, either it opens upward or downward, the given illustration is an example of upward parabola. Study the given illustrative examples as follows to know how to construct parabola and determine its domain, range, intercepts, axis of symmetry, vertex, and its

Study the following.

ILLUSTRATIVE EXAMPLES:

1. Given: $y = -2x^2$

Domain	Set of all real numbers
Range	$\{y/y \leq 0\}$
Opening of the parabola	Downward
Vertex	(0,0)
Axis of Symmetry	$x = 0$
x - intercept	$y = -2x^2 \rightarrow 0 = -2x^2$ $x = \{0\}$
y - intercept	$y = -2x^2 \rightarrow y = -2(0)^2$ $y = \{0\}$

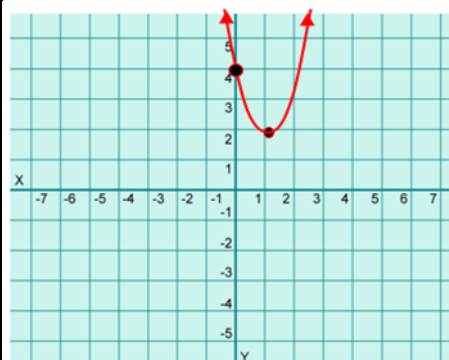


2. Given: $y = 2x^2 - 4x + 4$

Transform into $y = a(x - h)^2 + k$ (Vertex Form).

$$y = 2x^2 - 4x + 4 \rightarrow y = 2(x-1)^2 + 2$$

Domain	Set of all real numbers
Range	$\{y/y \geq 2\}$
Opening of the parabola	Upward
Vertex	(1, 2)
Axis of Symmetry	$x = 1$
x - intercept	$y = 2x^2 - 4x + 4 \rightarrow 0 = 2x^2 - 4x + 4$ no x- intercept
y - intercept	$y = 2x^2 - 4x + 4 \rightarrow y = 2(0)^2 - 4(0) + 4$ $y = \{4\}$



Note: To find the vertex in standard form, we can also use the formula for

$$(h,k): \mathbf{h} = \frac{-b}{2a}; \mathbf{k} = \frac{4ac - b^2}{4a}$$

In the function $y = 2x^2 - 4x + 4$; $a = 2$; $b = -4$; $c = 4$. Substitute the values to the formula and solve then simplify.

$$\text{Thus, } \mathbf{h} = \frac{-(-4)}{2(2)} = \frac{4}{4} = \mathbf{1} ;$$

$$\mathbf{k} = \frac{4(2)(4) - (-4)^2}{4(2)} = \frac{32 - 16}{8} = \frac{16}{8} = \mathbf{2} ; \mathbf{(1, 2) - \text{Vertex}}$$

The **domain** of quadratic function is the set of real numbers. In the form $y = a(x - h)^2 + k$, if $a > 0$, then the **range** of the quadratic function is $\{y/y \geq k\}$; if $a < 0$, then the range of the quadratic function is $\{y/y \leq k\}$.

The value of “a” indicates the **opening of the parabola**, if the value of “a” is positive (+) then the parabola opens upward and if the value of “a” negative (-) it opens downward.

The table below indicates the different forms of quadratic functions and its properties.

FORM	VERTEX	AXIS OF SYMMETRY
$y = ax^2$	(0,0)	$x = 0$
$y = ax^2 + k$	(0,k)	$x = 0$
$y = a(x-h)^2 + k$	(h,k)	$x = h$
$y = ax^2 + bx + c$	(h,k)	$x = h$

The **vertex** of the parabola is the point (h,k). It is the minimum point of the parabola if $a > 0$ and maximum point of the parabola if $a < 0$.

The **axis of symmetry** of the parabola is the vertical line $x = h$ and also divides the parabola into two equal parts.

The **x-intercepts** is determined by setting $y = 0$, and then solving for x. On the other hand, the **y - intercept** is determined by setting $x = 0$, and solve for y. The function $y = ax^2 + bx + c$ has two distinct x - intercepts if $b^2 - 4ac > 0$; only one x - intercept if $b^2 - 4ac = 0$; and no x - intercepts if $b^2 - 4ac < 0$.

Learning Task 3. Complete the tables below and graph on the same coordinate plane. Then analyze the relationship between the graphs.

1. Given: $y = -2x^2$

Domain	
Range	
Opening of the parabola	
Vertex	
Axis of Symmetry	
x - intercept	
y - intercept	

2. Given: $y = -x^2 + 4$

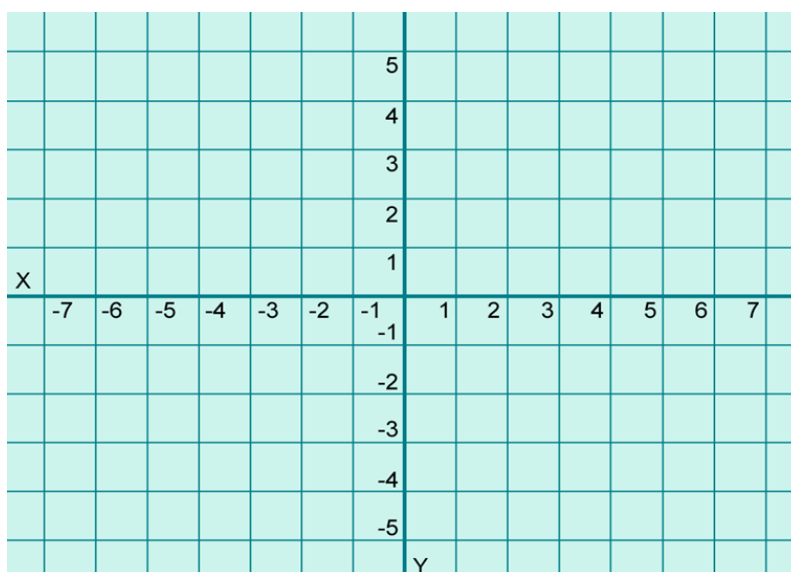
Domain	
Range	
Opening of the parabola	
Vertex	
Axis of Symmetry	
x - intercept	
y - intercept	

3. Given: $y = (x + 1)^2$

Domain	
Range	
Opening of the parabola	
Vertex	
Axis of Symmetry	
x - intercept	
y - intercept	

4. Given: $y = 2x^2 - 4x + 4$

Domain	
Range	
Opening of the parabola	
Vertex	
Axis of Symmetry	
x - intercept	
y - intercept	



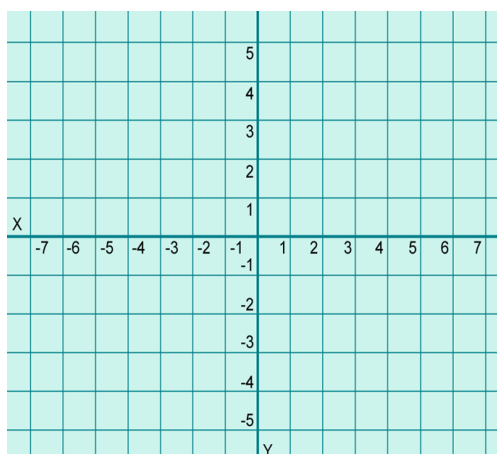
B. Show the following effects of the changing values of a , h and k in the equation $y = a(x - h)^2 + k$ of a quadratic function by formulating your own quadratic functions and graphing it.

- The parabola opens upward if $a > 0$ (positive) and opens downward if $a < 0$ (negative).
- The graph of $y = ax^2$ narrows if the value of “ a ” becomes larger and widens when the value of “ a ” is smaller. Its vertex is always located at the origin $(0, 0)$ and the axis of symmetry is $x = 0$.
- The graph of $y = ax^2 + k$ is obtained by shifting $y = ax^2$, k units upward if $k > 0$ (positive) and $|k|$ units downward if $k < 0$ (negative). Its vertex is located at the point of $(0, k)$ and an axis of symmetry of $x = 0$.
- The graph of $y = a(x - h)^2$ is obtained by shifting $y = ax^2$, h units to the right if $h > 0$ (positive) and $|h|$ units to the left if $h < 0$ (negative). Its vertex is located at the point of $(h, 0)$ and an axis of symmetry of $x = h$.
- The graph of $y = a(x - h)^2 + k$ is obtained by shifting $y = ax^2$, h units to the right if $h > 0$ (positive) $|h|$ units to the left if $h < 0$ (negative).; and k units upward if $k > 0$ (positive) and $|k|$ units downward if $k < 0$ (negative). Its vertex is located at the point of (h, k) and an axis of symmetry of $x = h$.



Learning Task 4. Illustrate the graphs of the following quadratic functions then analyze the effects from each other.

1. $y = 3x^2$
2. $y = 3x^2 - 3$
3. $y = 3x^2 + 3$
4. $y = 3(x - 3)^2 + 1$
5. $y = 3(x + 3)^2 - 1$



Explanation:

Graphing Quadratic Functions and Analyzing the Effects on its Graph

I

After going through this module, you are expected to:

- Determines the equation of a quadratic function given: (a) a table of values; (b) graph; (c) zeros.
- Solves problems involving quadratic functions.

Complete the table using the function of x .

x	-3	-2	-1	0	1	2	3
$f(x) = 3x^2 - 5$							

D

Equation of the Quadratic function can be determine through table.

Illustrative Example 1

x	1	2	3	4	5	6	7
y	5	11	19	29	41	55	71

1st Differences 6 8 12 14 16 18

2nd Differences 2 2 2 2 2

Solution: Let the quadratic function f be of the form $y = ax^2 + bx + c$ where a , b and c are to be determined. Let us consider any 3 ordered pairs (x, y) from the table.

$$\text{Equation 1} \rightarrow 5 = a(1)^2 + b(1) + c \rightarrow 5 = a + b + c$$

$$\text{Equation 2} \rightarrow 19 = a(3)^2 + b(3) + c \rightarrow 19 = 9a + 3b + c$$

$$\text{Equation 3} \rightarrow 29 = a(4)^2 + b(4) + c \rightarrow 29 = 16a + 4b + c$$

We obtain a systems of linear equations in 3 unknowns a , b and c .

$$(1) 5 = a + b + c$$

$$(2) 19 = 9a + 3b + c$$

$$(3) 29 = 16a + 4b + c$$

– (1) gives $8a + 2b = 14$ or $4a + b = 7$ (4)

– (2) gives $7a + b = 10$ (5)

(5) - (4) gives $3a = 3$ or $a = 1$

Substituting $a = 1$ in (4) yields $b = 3$.

Substituting $a = 1$ and $b = 3$ in (1), we obtain $c = 1$.

Thus, the quadratic function determined by the table is $y = x^2 + 3x + 1$.

Equation of the Quadratic function can also be determine backwards:

Illustrative Example 2

Determine an equation that has the solutions $x = -4$ and $x = 3$.

Work backward to find the equation:

$x = -4$	$x = 3$	given
$x + 4 = 0$	$x - 3 = 0$	set equal to 0
$(x + 4)(x - 3) = 0$		equation factor
$x^2 + x - 12 = 0$		product of factors

The equation is $x^2 + x - 12 = 0$

Illustrative Example 3

Find the solutions for the equation

$$3x^2 + 3x - 36 = 0$$

The equation can be solved by factoring

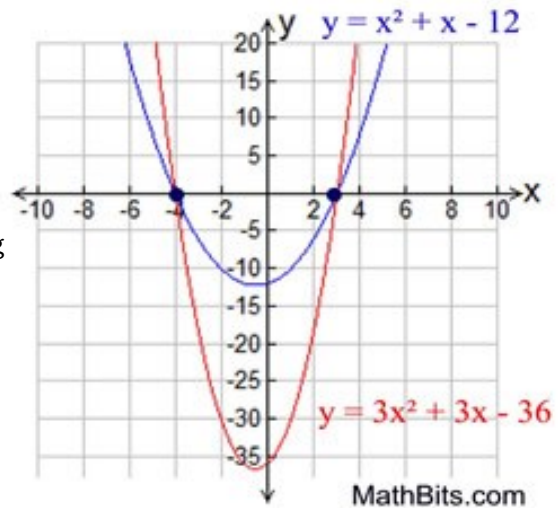
$$3x^2 + 3x - 36 = 0$$

$$3(x^2 + x - 12) = 0$$

$$3(x + 4)(x - 3) = 0$$

$$3 \neq 0; x + 4 = 0; x - 3 = 0$$

$$x = -4 \quad x = 3$$

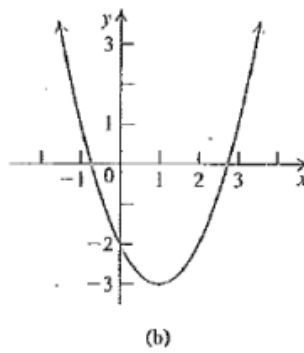
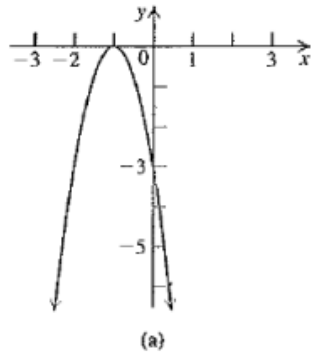


Note: Working backward will create an equation, but remember that there are other equations that will also have that same set of solutions.

Illustrative Example 4

If the x-intercepts are $(-3, 0)$ and $(4, 0)$, we know that the roots (zeros) of the equation will be $x = -3$ and $x = 4$. Working backward, we can create the factors $(x + 3)$ and $(x - 4)$ and get the equation $y = (x + 3)(x - 4)$. Then the equation is $y = x^2 - x - 12$.

Find the equation of the parabolas below. Put your answer in standard form.



E

The sum of two numbers is 29. Find the maximum possible product of the two numbers.

Solution:

Let x be the first number

$29 - x$ be the second number

y be the maximum possible product of the two numbers

$$y = x(29 - x)$$

$$y = 29x - x^2$$

The graph of $y = 29x - x^2$ opens downward and has a maximum value which is equal to **k** and it occurs at **h**.

Solving for the value of k ,

$$k = \frac{4ac - b^2}{4a} = \frac{4(-1)(0) - 29^2}{4(-1)} = 210.25$$

The two numbers that will give its maximum possible product are equal to h .

$$h = -\frac{b}{2a} = -\frac{29}{2(-1)} = 14.5$$

Learning Task 2. Solve each problem using quadratic functions.

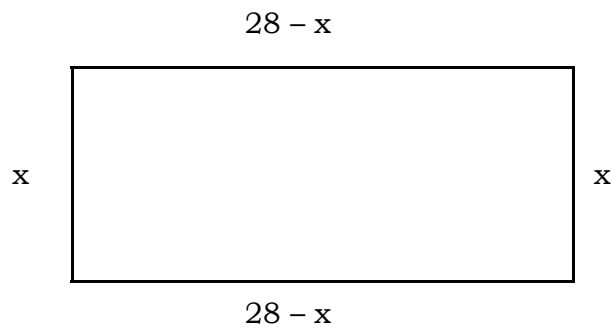
1. The sum of two positive integers is 35. What is the minimum sum of their squares?
2. A rectangle has a perimeter of 100 cm. Find the greatest possible area for the rectangle.

Find the quadratic function determined by each table.

x	-2	-1	0	1	2	3
y	4	0	-2	-2	0	4



Learning Task 3. Find the largest area which the farmer can enclose with 56 m of fencing materials.





Reference

Learner's Material for Mathematics Grade 9 (2013) Module 1: Quadratic Equations and Inequalities

Learner's Material for Mathematics Grade 9 (2013) Module 2: Quadratic Functions

Oronce & Mendoza (2016), e-Math Worktext in Mathematics 9. Rex Publishing, Inc., Manila.

Synergy for Success in Mathematics 9, Singapore Approach (2016), SAP Education, e-Books Interactive Systems, Inc.

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